

Exam III , MTH 221 , Summer 2011

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QUESTION 1. (**Each = 2 points, Total 48points + Extra 2points = 50 points**) Circle the correct letter.

(i) One of the following is a linear transformation

a) $T : R^3 \to R^2, T(a_1, a_2, a_3) = (-a_2, 0, 1 + a_1)$ b) $T : R^3 \to R^2, T(a_1, a_2, a_3) = (a_1 + 2a_3, -5a_2)$

c) $T : R^3 \to R^2, T(a_1, a_2, a_3) = (-a_1, 2a_2 + a_3^2)$ d) None of the previous is correct

(ii) One of the following is a subspace of $R_{2\times3}$:

a) $D = \{\begin{bmatrix} a & -b & 2a+b \\ 0 & 3a & -4a+b \end{bmatrix} \mid a, b \in \text{R}\}$ b) $D = \{\begin{bmatrix} 0 & -b & 2a+b \\ 0 & 3a & -a+b \end{bmatrix} \mid a, b \geq 0\}$

c) $D = \{\begin{bmatrix} a & 3 & 2a+b \\ 0 & 3a & -4a+b \end{bmatrix} \mid a, b \in \text{R}\}$ d) None of the previous is correct.

(iii) Let $D = \{\begin{bmatrix} -a & b \\ 3a & -3b \end{bmatrix} \mid a, b \in R\}$ be a subspace of $R_{2\times2}$. Then a basis for D is :

a) $\{\begin{bmatrix} -1 & 0 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}\}$ b) $\{\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}\}$

c) $\{\begin{bmatrix} -1 & 0 \\ 3 & 0 \end{bmatrix}\}$ d) none of the previous is correct.

(iv) Let $T : R^2 \to R$ be a linear transformation such that $T(2, 0) = 4$ and $T(-2, 1) = 5$. Then $T(2, 1) =$

a) 9 b) 13 c) -1 d) can not be determined, more information is needed

(v) Let T as in the previous question. One of the following points belongs to $Ker(T)$:

a) $(-8, 4)$ b) (8, -4) c) (-18, 4) d) none of the previous is correct

(vi) Let $T : R^2 \to R^2, T(1, 0) = (1, 1)$ and $(0, 3) \in Ker(T)$. Then $Ker(T) =$

a) $\{(x_2, x_2) \mid x_2 \in R\}$ b) $\{(x_1, -3x_1) \mid x_1 \in R\}$ c) $\{(0, x_2) \mid x_2 \in R\}$ d) none of the previous is correct

(vii) Let T as above. Then $Range(T) =$

a) $span\{(1, 1), (0, 3)\}$ b) $span\{(1, 1)\}$ c) $span\{(1, 0), (0, 3)\}$ d) None of the previous is correct

(viii) Let $K = \{\begin{bmatrix} 2a+b & 4a+2b \\ c & c \end{bmatrix} \mid a, b, c \in R\}$ be a subspace of $R_{2\times2}$. Then $dim(K) =$

a) 2 b)3 c)1 d) None of the previous is correct

(ix) Let $T : R^2 \to R^3$ be a linear transformation such that $T(a_1, a_2) = (0, a_1 + a_2, -2a_1 - 2a_2)$. Then $T(3, 0) =$

a) (0, 3, -6) b) (0, 0, 0) c) (3, 3, 0) d)(0, 3, 0)

(x) Let T as above. The standard matrix representation of T is

a) $\begin{bmatrix} 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -2 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 1 & -2 \\ 1 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$

(xi) Let T as above. Then $Ker(T) =$

a) $span\{(0, 1)\}$ b) $span\{(1, 0)\}$ c) $span\{(-1, 1)\}$ d) none of the previous is correct

(xii) Let T as above. Then $Range(T) =$

a) $span\{(1, 1)\}$ b) $span\{(0, 1, 0)\}$ c)$span\{(0, 1, -2)\}$ d) None of the previous is correct

(xiii) Let A be a 3×2 such that Rank(A) = 2. Then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

a) has exactly one solution, $x_1 = x_2 = 0$. b) has infinitely many solutions

(xiv) All the below are linear transformation. Exactly one of them has no possibility of being ONTO:

a)$T : R^3 \to R^5$ b) $T : P_3 \to R$ c) $T : R^6 \to R^2$ d) $T : R_{2\times2} \to R^3$

(xv) All the below are linear transformation. Exactly one of them has no possibility of being 1-1 (one to one):

a)$T : R^3 \to R^4$ b) $T : P_3 \to R^2$ c) $T : R^6 \to R^9$ d) $T : R_{2\times2} \to R^5$

(xvi) Let A be an $n \times 4$ matrix such that rank(A) = 4. Then

a) $n = 4$ b) $n \geq 4$ c) $n < 4$ d) None of the previous is correct.

(xvii) Let $T : R^3 \to R^3$ be a linear transformation such that T is 1-1. Let M be the standard matrix representation of T. Then

a)$det(M) \neq 0$ b) T is ONOT c) (a) and (b) are correct d)none of the previous is correct

(xviii) Let $T : R^5 \to R^5$ be a linear transformation and let M be the standard matrix representation of T. Given M is invertible. Then

a) T is 1-1 b) Range(T) is a subspace of R^5 but not equal to R^5. c) T is not ONTO d) (b) and (c) are correct

(xix) Let $T : P_3 \to P_3$ such that $T(p(x)) = p'(x)$. We know T is linear transformation. Range (T) =

a) P^2 b) R c)P_3 d) none of the previous is correct

(xx) Let $T : R^5 \to R^7$ be a linear transformation and let M be the standard matrix representation of T. Given Rank(M) = 5. Then

a) $Ker(T) = \{(0, 0, 0, 0, 0)\}$ b) $Range(T) = R^7$ c) Every 5 independent points in R^7 form a basis for Range(T). d) none of the previous is correct

(xxi) Let $T : P_2 \to R$ be a linear transformation such that $T(1) = T(x) = 1$. Then $ker(T) =$

a) $span\{-1\}$ b) $span\{1 - x\}$ c) $\{0\}$ d) None of the previous is correct

(xxii) Let A be a 4×7 matrix. Then dim (N(A))

a) 4 b) ≥ 3 c) is at most 3 d) None is correct

(xxiii) Let $T : P_3 \to R_{2\times2}$ be a linear transformation such that $T(a_0 + a_1x + a_2x^2) = \begin{bmatrix} a_1 & a_1 \\ a_1 & 0 \end{bmatrix}$. Then Range(T) =

a) $span\{\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}\}$ b) $span\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\}$ c) $span\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\}$ d) None is correct

(xxiv) Let T as above. $Ker(T) =$

a) R b) $span\{1, x^2\}$ c) P_2 d) None is correct

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